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DEPARTMENT OF CIVIL ENGINEERING
AND ENGINEERING MECHANICS

THE PHOTOELASTIC DETERMINATION OF STRESS ON TRANSVERSE PLANES OF SYMMETRY FOR THE GENERAL AXI-SYMMETRIC CASE

by

E. A. FOX

Office of Naval Research Project NR-064-388
Contract Nonr-266(09)
Technical Report No. 15
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Abstract

The method of O'Rourke and Saenz of treating the gross retardation patterns of symmetrically strained cylinders and spheres as an Abel integral equation is combined with one scattered light measurement to provide a complete solution on transverse planes of symmetry for the general axi-symmetric problem. A simple expression is derived for three-dimensional "notch stresses."

I. Introduction

The standard three-dimensional photoelastic techniques: freezing-slicing, and scattered light probing, have intrinsic limitations. Slicing is destructive of the model, probing requires a multiplicity of measurements, and neither is easily adapted to dynamic loading.

The idea of determining the interior stresses from the integrated relative retardation pattern is tantalizing and has been pursued by several investigators. Poritsky[1] achieved a solution for cylindrical bars in a state of plane strain; Read,[2] a solution for cylindrical glass bulbs under restrictive conditions. Kammerer[3] established the integral equation for the relative retardation in the axi-symmetric case from Neumann's[4] equations.

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[1] H. Poritsky, Physics 5, 406-411 (1934).
[2] W. T. Read, Jr., J. Appl. Phys. 21, 250-257 (1950).
[3] A. Kammerer, Recherches sur la photoelasticimetrie (Hermann et Cie., Paris, 1944) pp. 141, 157.
[4] F. E. Neumann, Abh. d. Kon. Akad. d. Wissenschaften zu Berlin (1841) Part II, p. 1.

O'Rourke and Saenz[5] observed that this equation is Abel's integral equation. They concerned themselves with axially symmetric plane strain in long cylinders and radially symmetric stress in spheres, but required a very restrictive sum rule, $\sigma_z = \sigma_r + \sigma_\theta$, in the case of cylinders to obtain a solution. This restriction was removed by Saenz[6] by interferometric data and by Drucker and Woodward[7] by the use of oblique incidence.

The following discussion will apply to the general axi-symmetric case on a plane of transverse symmetry of an elastic body.

II. Preliminary Equations

Let the normal to the wave front be parallel to the y-axis, then the Maxwell-Neumann stress optic law[8] relates the relative retardation δ, the principal stresses in the plane of the wave front p, q $(p>q)$, the orientation ϕ of q with respect to x, and γ, the arc tan of the amplitude ratio of the two transmitted waves, as follows:

$$\frac{\partial \delta(x,y,z)}{\partial y} + C(p-q) = 2\frac{\partial \phi}{\partial y}\cot 2\gamma \sin\delta$$

$$\frac{\partial \gamma}{\partial y} = -\frac{\partial \phi}{\partial y}\cos\delta \tag{1}$$

where C is the stress-optic coefficient.

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[5] R. C. O'Rourke and A. W. Saenz, Quart. Appl. Math. 8, 303-311 (1950). R. C. O'Rourke, J. Appl. Phys. 22, 872-878 (1951).
[6] A. W. Saenz, J. Appl. Phys. 21, 962-965 (1950).
[7] D. C. Drucker and W. B. Woodward, J. Appl. Phys. 25, 510-512 (1954).
[8] E. G. Coker and L. N. G. Filon, A Treatise on Photo-Elasticity (Cambridge University Press, Cambridge, 1931) p. 256.

Let the axis of symmetry be z. Then, in a usual notation, the following relations[9] hold:

$$\frac{\partial \sigma_r}{\partial r} + \frac{\partial \tau_{rz}}{\partial z} + \frac{\sigma_r - \sigma_\theta}{r} = 0 \tag{2}$$

$$\frac{\partial \sigma_z}{\partial z} + \frac{\partial \tau_{rz}}{\partial r} + \frac{1}{r}\tau_{rz} = 0 \tag{3}$$

$$\frac{\partial u}{\partial r} = \epsilon_r = \frac{1}{E}\left[\sigma_r - \nu(\sigma_\theta + \sigma_z)\right] \tag{4}$$

$$\frac{u}{r} = \epsilon_\theta = \frac{1}{E}\left[\sigma_\theta - \nu(\sigma_r + \sigma_z)\right] \tag{5}$$

where u is the displacement in the r direction. Eliminating u between (4) and (5) we obtain:

$$\frac{\sigma_r - \sigma_\theta}{r} = \frac{1}{1+\nu}\frac{\partial}{\partial r}\left[\sigma_\theta - \nu(\sigma_r + \sigma_z)\right] = \frac{E}{1+\nu}\frac{\partial}{\partial r}\left(\frac{u}{r}\right) \tag{6}$$

In the plane of the wave front

$$p - q = \left[\left(\sigma_r \frac{x^2}{r^2} + \sigma_\theta \frac{y^2}{r^2} - \sigma_z\right)^2 + 4\tau_{rz}^2 \frac{x^2}{r^2}\right]^{\frac{1}{2}} \tag{7}$$

$$\sin 2\phi = \frac{2\tau_{rz}}{p-q}\frac{x}{r} \tag{8}$$

III. Transverse Plane of Symmetry

Let $z = 0$ be a transverse plane of symmetry. Let a, b be the inner and outer radii, respectively, of the section of the body cut by $z = 0$. On $z = 0$, $\tau_{rz} = 0$. Hence, from (8), $\phi = 0$: therefore (1) and (7) become

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[9] A. E. H. Love, Mathematical Theory of Elasticity, 4th Ed., (Dover Publications, New York, 1944) p. 274.

$$\frac{\partial \delta(x,y,0)}{\partial y} = -C(p-q)_{z=0} = C\left[\sigma_z - \sigma_r \frac{x^2}{r^2} - \sigma_\theta \frac{y^2}{r^2}\right]_{z=0} \tag{9}$$

Consider a pencil of circularly polarized light along the path $x = 0$. Then (9) becomes

$$C(\sigma_z - \sigma_\theta)_{z=0} = \frac{\partial \delta(0,y,0)}{\partial y} = \frac{\partial \delta(r,0)}{\partial r} = S(r) \tag{10}$$

where $S(r)$ is Weller's[10] scattered light function which is inversely proportional to the spacing of the interference fringes viewed normally to the light path.

Let $R(x,z)$ be the two-dimensional map of the integrated relative retardation. Put (6) and (10) in (9) and integrate across the chord with respect to y. Let $t = [b^2 - x^2]^{\frac{1}{2}}$ be the half chord length. Then since all functions are even in y

$$R(x,0) = 2\int_0^t S(r)\,dy - 2C\frac{E}{1+\nu}x^2\int_0^t \frac{1}{r}\frac{d}{dr}\left(\frac{u(r,0)}{r}\right)dy$$

Changing the variable of integration to r and transposing, there results

$$\frac{1+\nu}{2CE}\frac{1}{x^2}\left\{2\int_x^b \frac{r\,S(r)}{\sqrt{r^2-x^2}}\,dr - R(x,0)\right\} = \int_x^b \frac{\frac{d}{dr}\left(\frac{u(r,0)}{r}\right)}{\sqrt{r^2-x^2}}\,dr \tag{11}$$

This is Abel's integral equation,[11] which, since the left hand side of (11) vanishes at $x = b$, has the unique continuous inverse

$$\frac{d}{dr}\left(\frac{u(r,0)}{r}\right) = \frac{1+\nu}{\pi EC}\frac{d}{dr}\int_r^b \frac{R(x,0) - 2\int_x^b \frac{r\,S(r)}{\sqrt{r^2-x^2}}\,dr}{x\sqrt{x^2-r^2}}\,dx = \frac{1+\nu}{\pi EC}\frac{d}{dr}M(r) \tag{12}$$

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[10] R. Weller, Nat. Adv. Comm. Aero. Tech. Note 737, 1939.
[11] E. T. Whittaker and G. N. Watson, Modern Analysis (Cambridge University Press, New York, 1945) p. 229.

where $M(r)$ is an experimentally determined function. Integrate (12) with respect to r. Then, since $M(b)=0$,

$$u(r,0) = r\left[\frac{1+\nu}{\pi E C} M(r) + \frac{u(b,0)}{b}\right] \tag{13}$$

where $u(b,0)$ is determined by measurement or is computed. (See Section IV.)

Sum (4) and (5), yielding

$$\sigma_r + \sigma_\theta = \frac{1}{1+\nu}\left[E\,\frac{1}{r}\frac{\partial}{\partial r}(ru) + 2\nu\sigma_z\right] \tag{14}$$

Finally, solving (6), (10), and (14)

$$\sigma_z(r,0) = \frac{E}{(1+\nu)(1-2\nu)}\left[\frac{u(r,0)}{r} + \nu\frac{du(r,0)}{dr}\right] + \frac{1-\nu}{C(1-2\nu)} S(r) \tag{15}$$

$$\sigma_\theta(r,0) = \frac{E}{(1+\nu)(1-2\nu)}\left[\frac{u(r,0)}{r} + \nu\frac{du(r,0)}{dr}\right] + \frac{\nu}{C(1-2\nu)} S(r) \tag{16}$$

$$\sigma_r(r,0) = \frac{E}{(1+\nu)(1-2\nu)}\left[2\nu\frac{u(r,0)}{r} + (1-\nu)\frac{du(r,0)}{dr}\right] + \frac{\nu}{C(1-2\nu)} S(r) \tag{17}$$

where $u(r)$ is given by (13), and $S(r)$ by (10).

IV. Determination of $u(b,0)$

$u(b,0)$ may either be measured or computed as follows: Put (2) in (9) and integrate with respect to y.

$$\frac{1}{2C} R(x,0) = \int_0^t \left[\sigma_z - \sigma_r - \frac{\partial\sigma_r}{\partial r}\frac{y^2}{r} - \frac{\partial\tau_{rz}}{\partial z}\frac{y^2}{r}\right]_{z=0} dy$$

Integrate the second term by parts, then after some manipulation (See Appendix.)

$$\frac{1}{2C} R(x,0) = \int_x^b \frac{\sigma_z(r,0) - \sigma_r(b,0) - \int_r^b \frac{\partial\tau_{rz}(r,0)}{\partial z} dr}{\sqrt{r^2 - x^2}}\, r\, dr \tag{18}$$

This is again Abel's integral equation with the unique inverse

$$\sigma_z(r,0) = \sigma_r(b,0) - \frac{1}{\pi C}\frac{1}{r}\frac{d}{dr}\int_r^b \frac{R(x,0)}{\sqrt{x^2-r^2}}\, x\,dx + \int_r^b \frac{\partial \tau_{rz}(r,0)}{\partial z}\,dr \tag{19}$$

Put (2) in (6), then

$$\frac{\partial}{\partial r}(\sigma_r + \sigma_\theta) = \nu\frac{\partial \sigma_z}{\partial r} - (1+\nu)\frac{\partial \tau_{rz}}{\partial z} \tag{20}$$

Integrate (20) with respect to r. Then this with (5), (14), and (19) yields

$$E\frac{1}{r}\frac{d}{dr}\left[r\,u(r,0)\right] = E(1-\nu)\frac{u(b,0)}{b} + (1-\nu^2)\frac{1}{\pi C}\frac{1}{r}\frac{d}{dr}\int_r^b \frac{R(x,0)}{\sqrt{x^2-r^2}}\,x\,dx$$

$$+ (1+\nu)(1-2\nu)\,\sigma_z(r,0)$$

or, integrating,

$$b\,u(b,0) - a\,u(a,0) = \frac{1-\nu}{2}\,\frac{u(b,0)}{b}(b^2-a^2) - \frac{1-\nu^2}{\pi E C}\int_a^b \frac{R(x,0)}{\sqrt{x^2-a^2}}\,x\,dx$$

$$+ \frac{(1+\nu)(1-2\nu)}{E}\,\frac{P}{2\pi} \tag{21}$$

where $P = 2\pi\int_a^b \sigma_z(r,0)\,r\,dr$ = total axial force across $z = 0$.

Put (13) in (21), obtaining

$$u(b,0) = \frac{2b}{(1+\nu)(b^2-a^2)}\left[a^2\frac{1+\nu}{\pi E C}M(a) - \frac{1-\nu^2}{\pi E C}\int_a^b \frac{R(x,0)}{\sqrt{x^2-a^2}}\,x\,dx + \frac{(1+\nu)(1-2\nu)}{E}\,\frac{P}{2\pi}\right] \tag{22}$$

If in particular $a = 0$, then

$$u(b,0) = \frac{1}{b\pi E}\left[(1-2\nu)P - \frac{2(1-\nu)}{C}\int_0^b R(x,0)\,dx\right] \tag{22'}$$

V. Alternate Expression for $\sigma_z(b,o)$

If it is desired to obtain only the axial stress on the outer boundary of the section $z = o$, then a simple expression may be derived directly from the Maxwell-Neumann law. Consider (9). Let $x \to b$. Then $r \to b$, $t \to y \to o$, $\delta(x,y,o) \to \delta(b,o,o) = \frac{1}{2} R(b,o)$. Replot $R(x,o)$ as $R(t,o)$ where $t = [b^2 - x^2]^{\frac{1}{2}}$. Then in the limit

$$\sigma_z(b,o) = \frac{1}{2C}\left[\frac{dR(t,o)}{dt}\right]_{t=o} + \sigma_r(b,o) \tag{23}$$

where $\sigma_r(b,o)$ = the normal surface traction at (b,o). Thus $\sigma_z(b,o)$ is proportional to the gradient at the boundary of the integrated relative retardation reckoned as a function of the half light path. We observe that (23) is consistent with (19) if R is made a function of t. Further, if $u(b,o)$ is known, then (5) and (23) yield

$$\sigma_\theta(b,o) = E\frac{u(b,o)}{b} + \nu\left\{2\sigma_r(b,o) + \frac{1}{2C}\left[\frac{dR}{dt}(t,o)\right]_{t=o}\right\} \tag{24}$$

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The author wishes to thank Professor R. D. Mindlin for suggesting this investigation and for his advice during its course.

Appendix

Derivation of Equation (18)

Integrate (9) with respect to y.

$$\frac{1}{2c} R(x,0) = \int_0^t \left[\sigma_z - \sigma_r \frac{x^2}{r^2} - \sigma_\theta \frac{y^2}{r^2}\right]_{z=0} dy = \int_0^t \left[(\sigma_z - \sigma_r) + (\sigma_r - \sigma_\theta)\frac{y^2}{r^2}\right]_{z=0} dy$$

Then, with (2),

$$\frac{1}{2c} R(x,0) = \int_0^t \left[\sigma_z - \sigma_r - \left(\frac{\partial \sigma_r}{\partial r} + \frac{\partial \tau_{rz}}{\partial z}\right)\frac{y^2}{r}\right]_{z=0} dy$$

Integrate the second term by parts, yielding

$$\frac{1}{2c} R(x,0) = \int_0^t \left[\sigma_z - \frac{\partial \tau_{rz}}{\partial z}\frac{y^2}{r}\right]_{z=0} dy - \Big[y\sigma_r(r,0)\Big]_0^t + \int_0^t \left[y \frac{\partial \sigma_r}{\partial r}\frac{dr}{dy}\right]_{z=0} dy$$

$$- \int_0^t \left[\frac{\partial \sigma_r}{\partial r}\frac{y^2}{r}\right]_{z=0} dy$$

but $\frac{dr}{dy} = \frac{y}{r}$ and $\Big[y\sigma_r(r,0)\Big]_0^t = \int_0^t \sigma_r(b,0)\, dy$, hence

$$\frac{1}{2c} R(x,0) = \int_0^t \left[\sigma_z - \sigma_r(b,0) - \frac{\partial \tau_{rz}}{\partial z}\frac{y^2}{r}\right]_{z=0} dy$$

Consider

$$\int_0^t \frac{\partial \tau_{rz}}{\partial z}\frac{y^2}{r}\, dy = \int_0^t \left\{\frac{d}{dr}\int_b^r \frac{\partial \tau_{rz}}{\partial z} dr\right\}\frac{y^2}{r} dy = \int_0^t \frac{r}{y}\left\{\frac{d}{dy}\int_b^r \frac{\partial \tau_{rz}}{\partial z} dr\right\}\frac{y^2}{r} dy$$

$$= \left[y \int_b^r \frac{\partial \tau_{rz}}{\partial z} dr\right]_0^t - \int_0^t \left\{\int_b^r \frac{\partial \tau_{rz}}{\partial z} dr\right\} dy$$

Thus

$$\frac{1}{2c} R(x,0) = \int_0^t \left[\sigma_z - \sigma_r(b,0) - \int_r^b \frac{\partial \tau_{rz}}{\partial z} dr\right]_{z=0} dy$$

Then, changing the variable of integration to r, we obtain (18).

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